## related expressions & Problems

## **UNIT-II- AC Circuits**

- Derive the expression for RMS and Average value of Sine wave.
- Define i) form factor ii) Peak factor iii) phase iv) phase difference v) Line voltage vi) phase voltage vii) line current viii) phase current.
- 3. Draw the phasor diagram for AC through pure resistor, pure inductor and pure capacitor.
- 4. Derive the expression for Current, power factor and power in Series R-L circuit
- 5. Derive the expression for Current, power factor and power in Series R-C circuit.
- 6. Derive the expression for Current, power factor and power in Series R-L-C circuit
- 7. Explain the operation of a series RLC circuit, when excited by AC supply with neat diagram
- 8. Derive the voltage and current relations in star and delta connected systems.
- 9. Explain the two wattmeter method with neat diagram? And draw the phasor diagram.
- Define Active, Reactive and Apparent power in AC circuits. What do you understand by Balanced loads.

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Derive Average & Rms value of Sinusoidal /Ac waveform

Average value

Voms = 
$$\frac{1}{T} \int_{0}^{T} V^{2}(t) dt$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} Vm^{2} \sin^{2} wt (dwt)$$

$$= \frac{Vm^{2}}{2\pi} \int_{0}^{2\pi} Sin^{2} wt dt$$

$$Voms = \frac{Vm^{2}}{2\pi} \left[ \frac{1-\cos 2(wt)}{2} \right]^{2\pi} = \frac{Vm}{\sqrt{2}}$$

#### 1. RMS Value of a Sine Wave

#### **Definition:**

The RMS value of a periodic waveform is the square root of the average of the square of the waveform over one full cycle.

$$V_{rms} = \sqrt{rac{1}{T} \int_0^T [v(t)]^2 dt}$$

Substitute  $v(t)=V_m\sin(\omega t)$ , and since a sine wave is periodic, we can take the limits over one period  $T=rac{2\pi}{\omega}$ :

$$V_{rms} = \sqrt{rac{1}{T}\int_0^T V_m^2 \sin^2(\omega t) dt} = V_m \sqrt{rac{1}{T}\int_0^T \sin^2(\omega t) dt}$$

Use identity:

$$\sin^2(\omega t)=rac{1-\cos(2\omega t)}{2}$$
  $V_{rms}=V_m\sqrt{rac{1}{T}\int_0^Trac{1-\cos(2\omega t)}{2}dt}=V_m\sqrt{rac{1}{2T}\int_0^T[1-\cos(2\omega t)]dt}$ 

Since  $\int_0^T \cos(2\omega t) dt = 0$  over one full period:

$$V_{rms} = V_m \sqrt{rac{1}{2T} \cdot T} = V_m \sqrt{rac{1}{2}}$$

**Final RMS Value:** 

$$V_{rms} = rac{V_m}{\sqrt{2}} pprox 0.707 V_m$$

## 2. Average Value of a Sine Wave (over Half Cycle)

#### Definition

The average value of a sine wave over **one full cycle is zero**, so we take it over **half cycle** (0 to  $\pi$ ):

$$egin{align} V_{avg} &= rac{1}{\pi} \int_0^\pi V_m \sin( heta) d heta \ &= rac{V_m}{\pi} \left[ -\cos( heta) 
ight]_0^\pi = rac{V_m}{\pi} \left[ -\cos(\pi) + \cos(0) 
ight] \ &= rac{V_m}{\pi} \left[ -(-1) + 1 
ight] = rac{V_m}{\pi} [2] \end{split}$$

Final Average Value (half cycle):

$$oxed{V_{avg} = rac{2V_m}{\pi} pprox 0.637V_m}$$

## Summary Table:

Quantity	Expression	Approx. Value
RMS Value	$\frac{V_{\mathrm{m}}}{\sqrt{2}}$	$pprox 0.707V_m$
Average Value	$\frac{2V_{\mathrm{m}}}{\pi}$	$pprox 0.637V_m$

## 2 Form Factor

#### **Definition:**

Form factor is the ratio of the RMS (Root Mean Square) value to the average value (mean value) of an alternating waveform.

$$\mathbf{Form}\,\mathbf{Factor} = \frac{\mathbf{RMS}\,\mathbf{Value}}{\mathbf{Average}\,\mathbf{Value}}$$

For a sine wave:

Form Factor = 
$$\frac{0.707V_m}{0.637V_m} = 1.11$$

## Peak Factor (also called Crest Factor)

#### **Definition:**

Peak factor is the ratio of the maximum (peak) value to the RMS value of an alternating waveform.

$$Peak Factor = \frac{Peak \ Value}{RMS \ Value}$$

For a sine wave:

$$\text{Peak Factor} = \frac{V_m}{0.707 V_m} \approx 1.414$$

## Phase

#### **Definition:**

Phase refers to the **position of a point** in time on a waveform cycle. It tells us the **timing** of a wave relative to a reference.

- Represented in degrees (°) or radians.
- · For example, if a sine wave starts at zero, it has 0° phase.

#### Phase Difference

#### **Definition:**

Phase difference is the **angular displacement** between two waveforms having the same frequency.

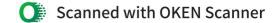
- . It indicates how much one wave leads or lags behind the other.
- · Expressed in degrees or radians.

Example: If Wave A leads Wave B by 90°, the phase difference is +90°.

## Line Voltage (in 3-phase systems)

#### **Definition:**

Line voltage is the voltage measured between any two lines (or phases) in a 3-phase system.



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For a star (Y) connection:

$$V_L = \sqrt{3} imes V_{ph}$$

## Phase Voltage

#### **Definition:**

Phase voltage is the voltage measured across a single phase (i.e., between a line and neutral) in a 3-phase system.

• In star connection, phase voltage is:

$$V_{ph}=rac{V_L}{\sqrt{3}}$$

• In delta connection, phase voltage = line voltage.

## Line Current

#### **Definition:**

Line current is the current flowing through each line conductor in a 3-phase system.

- In star connection, line current = phase current.
- In delta connection:

$$I_L = \sqrt{3} imes I_{ph}$$

## Phase Current

#### **Definition:**

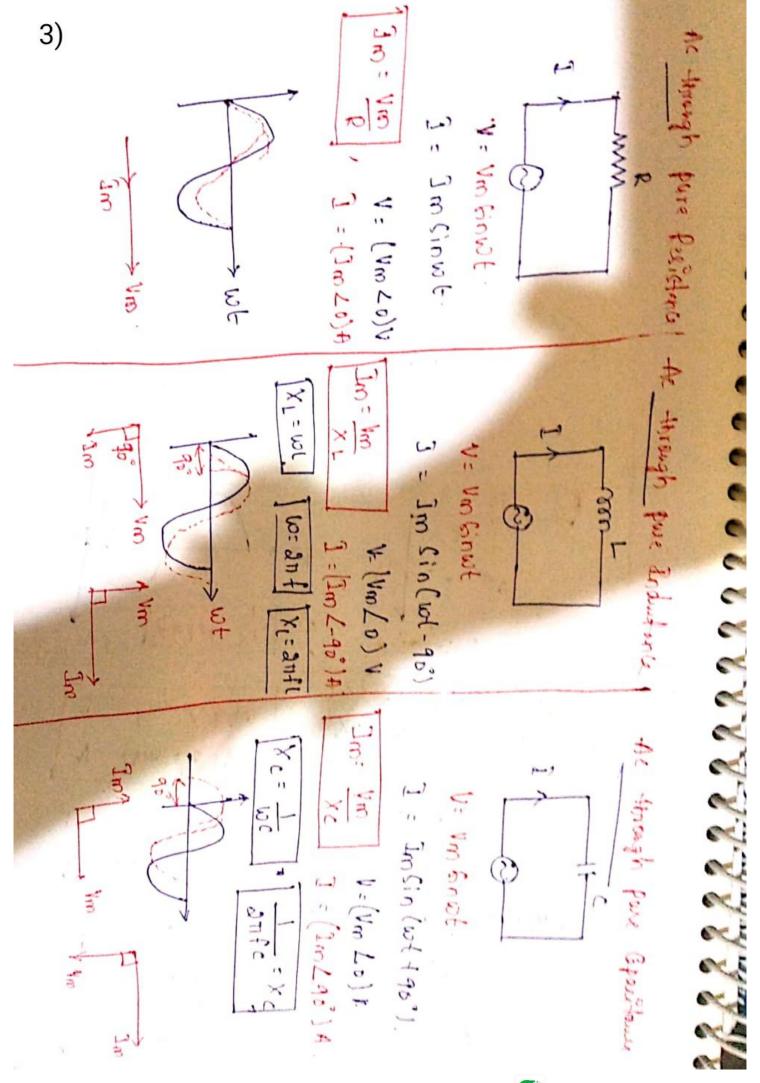
Phase current is the current flowing through each phase winding or load in a 3-phase system.

• In star connection:

$$I_{ph}=I_L$$

• In delta connection:

$$I_{ph}=rac{I_L}{\sqrt{3}}$$



## ◆ 4. Derive the expression for Current, Power Factor and Power in a Series R-L Circuit

• Given:

An AC supply is connected to a resistor (R) and inductor (L) in series.

Voltage equation:

$$V = V_R + V_L$$

Current is same through both:

$$I = \text{same in R and L (series)}$$

· Voltage across resistor:

$$V_R = IR$$

Voltage across inductor:

$$V_L = IX_L, \quad ext{where } X_L = \omega L$$

Total Impedance:

$$Z=\sqrt{R^2+X_L^2}$$

Expression for Current:

$$I = \frac{V}{\bar{Z}} = \frac{V}{\sqrt{R^2 + X_L^2}}$$

Power Factor:

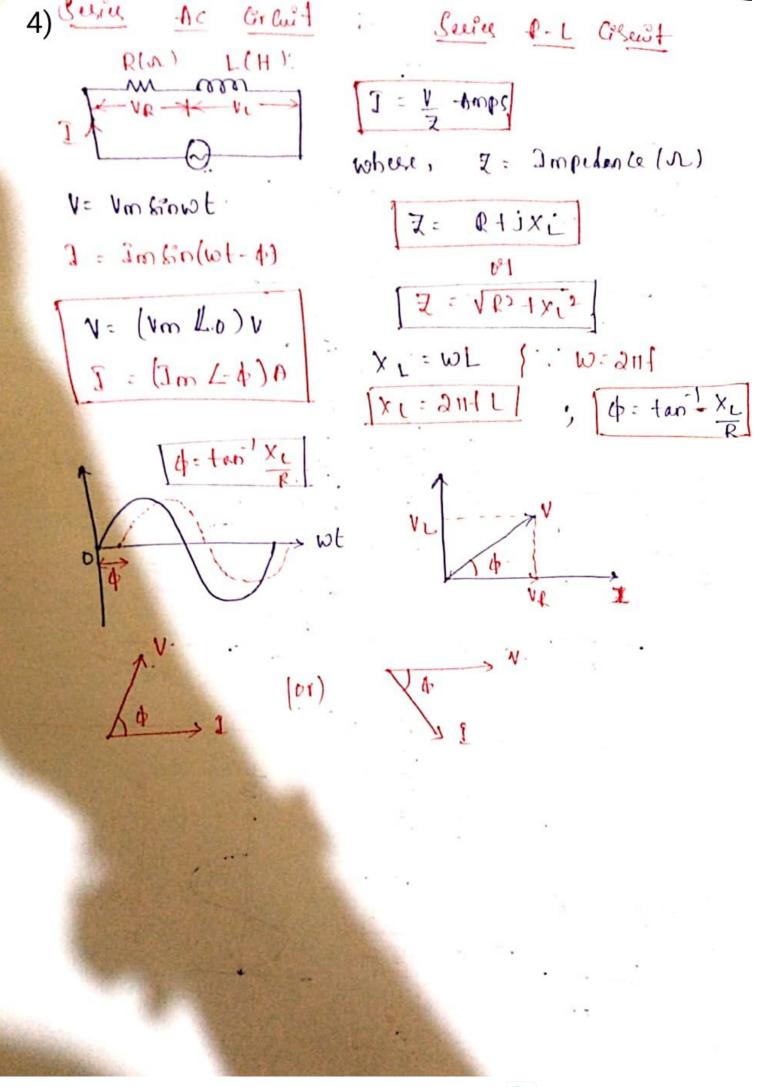
$$\cos\phi=rac{R}{Z}=rac{R}{\sqrt{R^2+X_L^2}}$$

Here,  $\phi$  is the phase angle between voltage and current.

Power:

$$P = VI\cos\phi = I^2R$$

Only resistor consumes power, inductor stores and returns it (no power loss).



# ◆ 5. Derive the expression for Current, Power Factor and Power in a Series R-C Circuit

Given:

An AC source is connected to resistor (R) and capacitor (C) in series.

Capacitive Reactance:

$$X_C = \frac{1}{\omega C}$$

Total Impedance:

$$Z=\sqrt{R^2+X_C^2}$$

Expression for Current:

$$I = rac{V}{Z} = rac{V}{\sqrt{R^2 + X_C^2}}$$

Power Factor:

$$\cos\phi=rac{R}{Z}=rac{R}{\sqrt{R^2+X_C^2}}$$

Current leads voltage in a capacitive circuit.

Power:

$$P = VI\cos\phi = I^2R$$

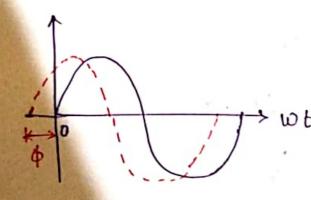
Only the resistor consumes power. Capacitor just stores and releases energy.

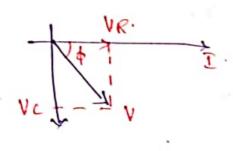
Series Ac Ormit

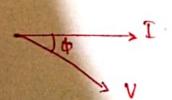
: Siere R-c cesuit

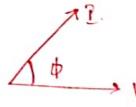
$$\widehat{I} = \frac{V}{Z} + mps$$

where ? = Impelance (12).









## 6. Derive the expression for Current, Power Factor and Power in Series R-L-C Circuit

- Components: Resistor (R), Inductor (L), Capacitor (C) in series with AC supply.
- Inductive Reactance:

$$X_L = \omega L$$

Capacitive Reactance:

$$X_C = \frac{1}{\omega C}$$

Net Reactance:

$$X = X_L - X_C$$

Total Impedance:

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

Expression for Current:

$$I=rac{V}{Z}=rac{V}{\sqrt{R^2+(X_L-X_C)^2}}$$

Power Factor:

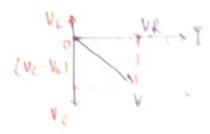
$$\cos\phi = rac{R}{Z}$$

- "If  $X_L>X_C$ : Inductive, current lags."
- "If  $X_C>X_L$ : Capacitive, current leads."
- ullet "If  $X_L=X_C$ : Resonance, PF = 1."

**V** Power:

$$P = VI\cos\phi = I^2R$$





## ◆ 7. Explain the Operation of Series RLC Circuit with AC Supply (with Neat Diagram)

Circuit Description:

A resistor R, inductor L, and capacitor C are connected in series to an AC source.

## **Operation:**

- . The total impedance (Z) determines the current.
- The circuit can behave resistive, inductive, or capacitive depending on the values of  $X_L$  and  $X_C$ .
- · The current varies based on the net reactance:

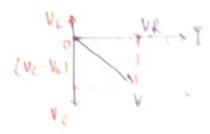
$$X = X_L - X_C$$

## Cases:

- 1. If  $X_L > X_C$ :
  - · Net reactance is inductive
  - Current lags voltage
  - · Power factor is lagging
- 2. If  $X_C > X_L$ :
  - · Net reactance is capacitive
  - · Current leads voltage
  - · Power factor is leading
- 3. If  $X_L = X_C$  (Resonance):
  - Net reactance = 0
  - Impedance is minimum = R
  - · Current is maximum
  - Power factor is unity (1)

## Phasor Diagram (at Resonance):





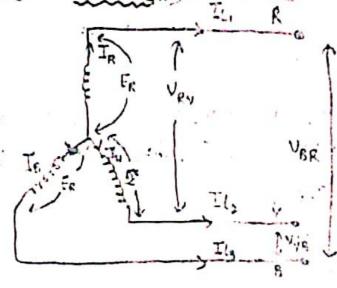
① 
$$I = \frac{V}{2} = \frac{230}{216.33} = 1.06A$$

RLC land 2= JR2+ (x1-x2)2 = J502+ (25,1-35.36)2 = 51.042

$$V = 230V$$
 $X_{c} = \frac{1}{20 \, \text{fc}} = \frac{1}{211 \, \text{x}} \frac{50 \, \text{x}}{90 \, \text{x}} \frac{10^{-1}}{10^{-1}} = 35.36 \, \text{g}$ 

$$T = \frac{V}{2} = \frac{230}{51.04} = 4.5A .$$

8) Stor connection (4). (4)

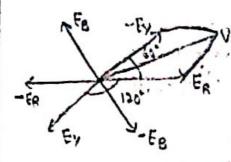


Il, Il, Ilz-line wosents = It IR. Iy, IB - phase wasents = Iph . ER, Ey, EB - phase voltages = Eph URY, UYB, VBR- line Voltages'= VL In y-connection

Apply KCL IR = IL, Iy = It2 IB = IL3.

In Y - connected gystern line current = phase current

Relation blw Is 8 Iph



VRY = ER - Ey RY VyB = Ey - EB VBA = EB - ER

VRY = JER + Ey + 2ER Ey COSO = JEph2+ Eph2+ 2 Eph Eph cos 60 = JEph2+ Eph2+25ph2 = J3Eph2

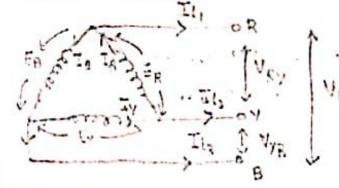
NRY = 53 Eph Vine = 53 phase voltage

In a 3 phase Star connected system having R= 150.0, C= 120 MF V= 420V and F= 50 Hz. find cost, IL, Iph, VL, Vph, P, D & S

D cost = R = 150 = 0.98 isode.

V<sub>eb</sub> = W<sub>L</sub> 420 Sua example problem

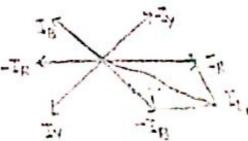




$$I_{L_1} = I_{R} - I_{R}$$

$$I_{L_2} = I_{Y} - I_{R}$$

$$I_{L_3} = I_{B} - I_{Y}$$



Sol: (1) cosp = 
$$\frac{R}{Z} = \frac{100}{100-44} = 0.99$$
 lagging
$$Z = \sqrt{R^2 + \pi_L^2} = \sqrt{100^2 + 9.42^2} = 100.44.$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 30 = 9.42.$$

# 9. Explain the Two Wattmeter Method with Neat Diagram. Draw the Phasor Diagram.

### Introduction:

The two wattmeter method is used to measure the power in a 3-phase system, especially when the system is balanced or unbalanced. It works for both star (Y) and delta ( $\Delta$ ) connected loads.

## Circuit Diagram:

(Please draw this diagram for your notes)

- Two wattmeters  $W_1$  and  $W_2$
- Current coils of both wattmeters are connected in Line 1 and Line 2
- Pressure (voltage) coils are connected between:
  - W<sub>1</sub>: Line 1 to Line 3
  - W2: Line 2 to Line 3
- · Load is connected in a 3-phase system.

## Working:

- Each wattrneter measures the power in its respective line.
- The total power in the 3-phase system is:

$$P_{total} = W_1 + W_2$$

## Phasor Diagram:

For a balanced inductive load, the current lags the voltage. The phasor diagram shows:

- Three phase voltages at 120° apart
- Current vectors lagging behind voltages by the power factor angle  $(\phi)$
- The angle between voltage across pressure coil and current through current coil determines wattrneter reading

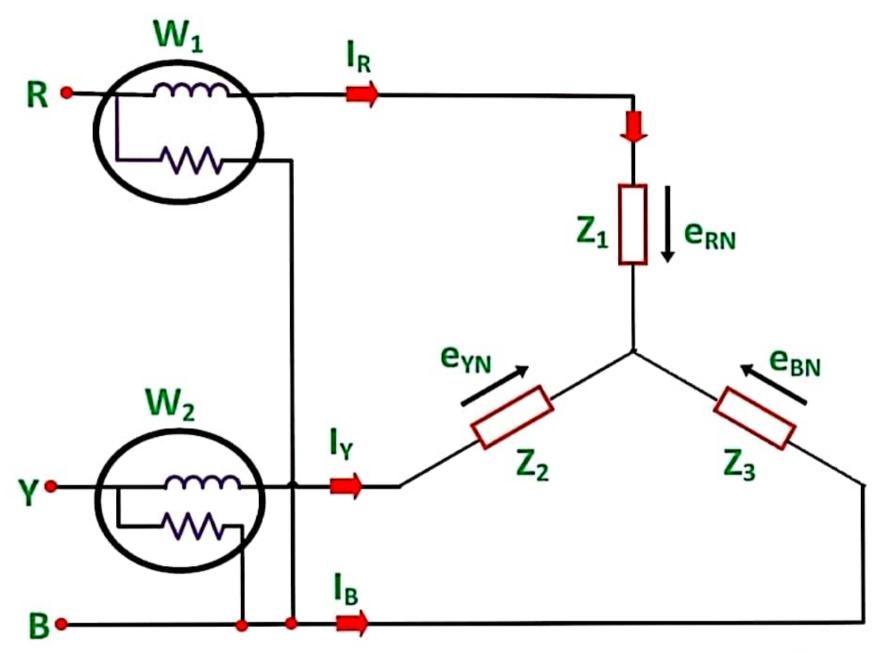
## **Conditions:**

- If power factor is unity, both wattmeters show equal reading.
- If power factor is lagging,  $W_1$  shows more than  $W_2$ .
- If one wattmeter reads negative, power factor < 0.5 (one needs to reverse connection to get a positive reading).
- Power factor angle φ can be found by:

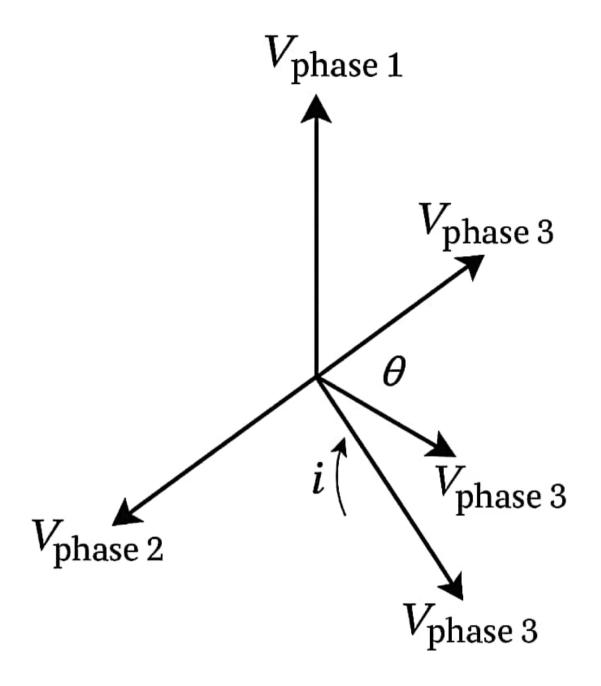
$$an\phi=rac{\sqrt{3}(W_1-W_2)}{W_1+W_2}$$

## Advantages:

- Only two wattmeters needed
- Works for both balanced and unbalanced loads



Circuit Globe



10. Define Active, Reactive, and Apparent Power in AC Circuits. What Do You Understand by Balanced Loads?

#### 1. Active Power (P):

- Also called Real Power or True Power
- It is the power actually consumed or used by the load to do useful work like lighting, heating, etc.
- Measured in Watts (W)
- Formula:

$$P = VI\cos\phi$$

where  $\cos \phi$  is the power factor.

#### 2. Reactive Power (Q):

- Power that flows back and forth between source and load.
- It does not do any useful work, but is needed for maintaining electric and magnetic fields in inductive or capacitive loads.
- Measured in VAR (Volt-Ampere Reactive)
- Formula:

$$Q = VI \sin \phi$$

#### 3. Apparent Power (S):

- · Total power supplied by the source to the circuit.
- · It is the vector sum of Active and Reactive power.
- · Measured in VA (Volt-Ampere)
- Formula:

$$S = VI$$

Or,

$$S=\sqrt{P^2+Q^2}$$

#### 4. Balanced Loads:

- · A load is said to be balanced when:
  - All three phases have equal impedance
  - The currents in all three lines are equal in magnitude and 120° apart
- In a balanced load:
  - · The system is symmetrical
  - Neutral current = 0
  - · Power calculations become simpler